Reg. No.____________ Name:________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FOURTH SEMESTER B.TECH DEGREE EXAMINATION, MAY 2017

MA202: PROBABILITY DISTRIBUTIONS, TRANSFORMS AND NUMERICAL METHODS

Max. Marks: 100 Duration: 3 Hours

Normal distribution table is allowed in the examination hall.

PART A (MODULES I AND II)

Answer two full questions.

1. a. Given that $f(x)=\frac{k}{2^x}$ is a probability distribution of a random variable that can take on the values $x=0,1,2,3$ and 4, find k. Find the cumulative distribution function. (7)

   b. If 6 of the 18 new buildings in a city violate the building code, what is the probability that a building inspector who randomly select 4 of the new buildings will catch
   - i) none of the new buildings that violate the building code
   - ii) one of the new buildings that violate the building code
   - iii) at least two of the new buildings violate the building code (8)

2. a. Prove that binomial distribution with parameters n and p can be approximated to Poisson distribution when n is large and p is small with $np=\lambda$ a constant. (7)

   b. Find the value of k for the probability density f(x) given below and hence find its mean and variance where

   $$f(x)=\begin{cases} kx^3 & 0<x<1 \\ 0 & otherwise \end{cases}$$ (8)

3. a. A random variable has normal distribution with $\mu=62.4$. Find it's standard deviation if the probability is 0.2 that it will take on a value greater than 79.2 (7)

   b. The amount of time that a surveillance camera will run without having to be reset is a random variable having the exponential distribution with the parameter 50 days. Find the probability that such a camera will
   - i) have to be reset in less than 20 days
   - ii) not have to be reset in at least 60 days. (8)

PART B (MODULES III AND IV)

Answer two full questions.

4. a. Use Fourier integral to show that $\int_0^\infty \frac{\cos x\omega+\omega\sin x\omega}{1+\omega^2}d\omega=\begin{cases} 0 & if\ x<0 \\ \pi/2 & if\ x=0 \\ \pi e^{-x} & if\ x>0 \end{cases}$ (7)

   b. Represent $f(x)=\begin{cases} x^2 & 0<x<1 \\ 0 & x>1 \end{cases}$ as a Fourier cosine integral. (8)

5. a. Find the Fourier transform of $f(x)=\begin{cases}1 & if\ |x|<1\\ 0 & otherwise\end{cases}$ (7)

b. Find the Laplace transforms of the following

i) $\cos t - t\sin t$ ii) $4t\,e^{-2t}$ (8)

6. a. Find the inverse Laplace transform of the following

i) $\dfrac{2s+1}{s^2+2s+5}$ ii) $\dfrac{(2s-10)}{s^3}e^{-5s}$ (8)

b. Solve $y''+2y'+5y=25t$, $y(0)=-2$, $y'(0)=-2$ using Laplace transforms (7)

PART C (MODULES V AND VI)

Answer two full questions.

7. a. Solve $f(x)=x-0.5\cos x=0$ near $x=0$ by fixed point iteration method. (7)

b. Solve $f(x)=2x-\cos x=0$ by Newton Raphson's method (7)

c. Find $f(9.2)$ from the values given below by Lagrange's interpolation formula

x	8	9	9.5	11
$f(x)$	2.197225	2.25 2.251292	2.397895	79442

(6)

8. a. Given $(x_j, f(x_j)) = (0.2, 0.$ 9980) ,(0.4,0.9686), (0.6, $0.8443), (0.8, 0.5358), (1,0)$, find $f(0.7)$ based on 0.2, 0.4, and 0.6 using newton's interpolation formula. (10)

b. Solve $10x_1+x_2+x_3=6,\ x_1+10x_2+x_3=6,\ x_1+x_2+10x_3=6$ by Gauss-Seidel iteration method starting at $x_1=1$, $x_2=1$ and $x_3=1$ correct to 4 digits. (10)

9. a. Evaluate $\int_0^1 \frac{1}{1+x^2}dx$ with 4 subintervals by Simpson's rule and compare it with the exact solution. (7)

b. Solve $y'=y$, $y(0)=1$ by Euler method to find $y(1)$ with $h=0.2$ (7)

c. Solve $y'=1+y^2$, $y(0)=0$ by fourth order Runge-Kutta method with $h=0.1$, 5 steps. (6)

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